# A.P. Physics 1 First Semester Review Sheet

Fall, Dr. Wicks

**Chapter 1: Introduction to Physics**

- Review types of zeros and the rules for significant digits
- Review mass vs. weight, precision vs. accuracy, and dimensional analysis problem solving.

**Chapter 2: One-Dimensional Kinematics**

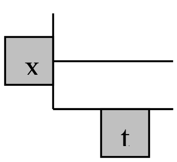
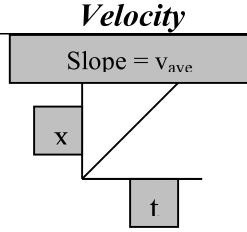
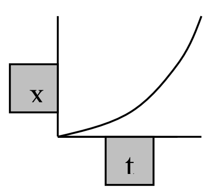
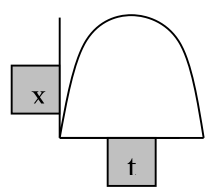
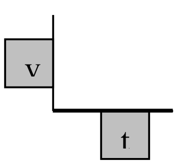
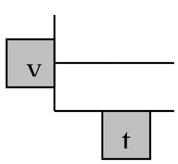
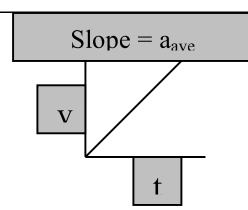
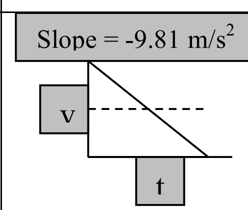
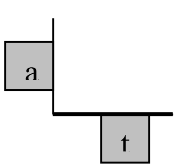
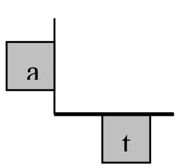
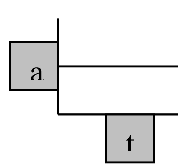
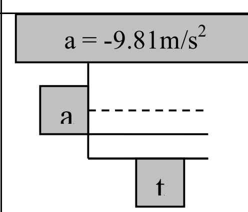
***A. Velocity***

- Equations for average velocity: $v_{ave} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ and $v_{ave} = \frac{1}{2}(v_f + v_i)$
- In a position-versus-time graph for constant velocity, the slope of the line gives the average velocity. See Table 1.
- Instantaneous velocity can be determined from the slope of a line tangent to the curve at a particular point on a position-versus-time graph.
- Use $v_{ave} = \frac{\Delta x_{Total}}{\Delta t_{Total}}$ to calculate the average velocity for an entire journey if given information about the various legs of the journey.

***B. Acceleration***

- Equation for average acceleration: $a_{ave} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$
- In a velocity-versus-time graph for constant acceleration, the slope of the line gives acceleration and the area under the line gives displacement. See Table 1.
- Acceleration due to gravity = $g$ = 9.81 m/s$^2$. (Recall $a$ = - $g$ = -9.81 m/s$^2$)

**Table 1: Graphing Changes in Position, Velocity, and Acceleration**

| | ***Constant Position*** | ***Constant Velocity*** | ***Constant Acceleration*** | ***Ball Thrown Upward*** |
|---|---|---|---|---|
| ***Position Versus Time:*** |  |  |  |  |
| ***Velocity Versus Time:*** |  |  |  |  |
| ***Accelera -tion Versus Time:*** |  |  |  |  |

| Table 2: Comparing the Kinematic Equations | |
|---|---|
| ***Kinematic Equations*** | ***Missing Variable*** |
| $x = x_o + v_{ave}t$ | $a$ |
| $v = v_o + at$ | $\Delta x$ |
| $x = x_o + v_o t + \frac{1}{2}at^2$ | $v_{final}$ |
| $v^2 = v_o{}^2 + 2a\Delta x$ | $\Delta t$ |

## Chapter 3: Vectors in Physics

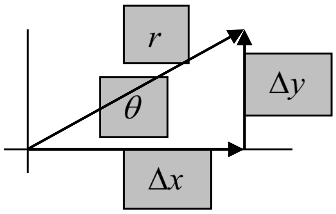
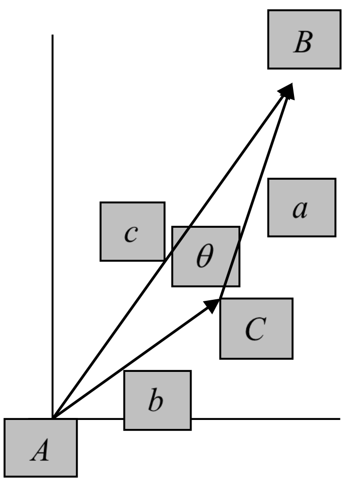
### *A. Vectors*

- ***Vectors*** have both magnitude and direction whereas ***scalars*** have magnitude but no direction.
- Examples of vectors are position, displacement, velocity, linear acceleration, tangential acceleration, centripetal acceleration, applied force, weight, normal force, frictional force, tension, spring force, momentum, gravitational force, and electrostatic force.
- Vectors can be moved parallel to themselves in a diagram.
- Vectors can be added in any order. See Table 3 for vector addition.
- For vector $\vec{r}$ at angle $\theta$ to the x-axis, the x- and y-components for $\vec{r}$ can be calculated from $\Delta x = r\cos\theta$ and $\Delta y = r\sin\theta$.
- The magnitude of vector $\vec{r}$ is $r = \sqrt{\Delta x^2 + \Delta y^2}$ and the direction angle for $\vec{r}$ relative to the nearest x-axis is $\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$.
- To subtract a vector, add its opposite.
- Multiplying or dividing vectors by scalars results in vectors.
- In addition to adding vectors mathematically as shown in the table, vectors can be added graphically. Vectors can be drawn to scale and moved parallel to their original positions in a diagram so that they are all positioned head-to-tail. The length and direction angle for the resultant can be measured with a ruler and protractor, respectively.

### *B. Relative Motion*

- Relative motion problems are solved by a special type of vector addition.
- For example, the velocity of object 1 relative to object 3 is given by $\vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23}$ where object 2 can be anything.
- Subscripts on a velocity can be reversed by changing the vector's direction: $\vec{v}_{12} = -\vec{v}_{21}$

**Table 3: Vector Addition**

| ***Vector Orientation*** | ***Calculational Strategy Used*** | |
|---|---|---|
| ***Vectors are parallel:*** | Add or subtract the magnitudes (values) to get the resultant. Determine the direction by inspection. | |
| ***Vectors are perpendicular:***   | Use the Pythagorean Theorem, $\Delta x^2 + \Delta y^2 = r^2$, to get the resultant, $r$, where $\Delta x$ is parallel to the x-axis and $\Delta y$ is parallel to the y-axis.<br><br>Use $\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right)$ to get the angle, $\theta$, made with the x-axis. | |
| ***Vectors are neither parallel nor perpendicular:*** | ***Adding 2 Vectors*** | ***Adding 2 or More Vectors (Vector Resolution Method)*** |
| | ***Limited usefulness*** | ***Used by most physicists*** |
|   | (1) Use the law of cosines to determine the resultant: $c^2 = a^2 + b^2 - 2ab\cos\theta$<br><br>(2) Use the law of sines to ***help*** determine direction: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ | (1) Make a diagram.<br>(2) Construct a vector table. (Use ***vector***, ***x-direction***, and ***y-direction*** for the column headings.)<br>(3) Resolve vectors using $\Delta x = r\cos\theta$ and $\Delta y = r\sin\theta$ when needed.<br>(4) Determine the signs.<br>(5) Determine the sum of the vectors for each direction, $\Delta x_{total}$ and $\Delta y_{total}$.<br>(6) Use the Pythagorean Thm to get the resultant, $r$: $\Delta x_{total}{}^2 + \Delta y_{total}{}^2 = r^2$<br>(7) Use $\theta = \tan^{-1}\left(\frac{\Delta y_{total}}{\Delta x_{total}}\right)$ to get the angle, $\theta$. |

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## Chapter 4: Two-Dimensional Kinematics

### *A. Projectile Motion*

- See Table 4 to better understand how the projectile motion equations can be derived from the kinematic equations.
- The kinematic equations involve ***one-dimensional motion*** whereas the projectile motion equations involve ***two-dimensional motion***. Two-dimensional motion means there is motion in both the horizontal and vertical directions.
  - Recall that the equation for horizontal motion (ex. $\Delta x = v_x \Delta t$) and the equations for vertical motion (ex. $v_{y,f} = -g\Delta t$, $\Delta y = -\frac{1}{2}g(\Delta t)^2$, $v_{y,f}^2 = -2g\Delta y$) are independent from each other.
  - Recall that velocity is constant and acceleration is zero in the horizontal direction.
  - Recall that acceleration is $g = 9.81$ m/s$^2$ in the vertical direction.
- When projectiles are launched at an angle, the ***range*** of the projectile is often calculated from $\Delta x = (v_i \cos\theta)\Delta t$ and its ***time of flight*** is often calculate from $\Delta y = (v_i \sin\theta)\Delta t - \frac{1}{2}g(\Delta t)^2$.
- Projectiles follow a ***parabolic pathway*** governed by $y = h - \left(\frac{g}{2v_o{}^2}\right)x^2$

**Table 4: Relationship Between the Kinematic Equations and Projectile Motion Equations**

| ***Kinematic Equations*** | ***Missing Variable*** | ***Projectile Motion, Zero Launch Angle*** | ***Projectile Motion, General Launch Angle*** |
|---|---|---|---|
| | | ***Assumptions made:*** $a = -g$ and $v_{o,y} = 0$ | ***Assumptions made:*** $a = -g$, $v_{o,x} = v_o \cos\theta$, and $v_{o,y} = v_o \sin\theta$ |
| $x = x_o + v_{ave}t$ | $a$ | $\Delta x = v_x t$ where $v_x = const.$ | $\Delta x = (v_o \cos\theta)t$ where $v_x = const.$ |
| $v = v_o + at$ | $\Delta x$ | $v_y = -gt$ | $v_y = v_o \sin\theta - gt$ |
| $x = x_o + v_o t + \frac{1}{2}at^2$ | $v_{final}$ | $\Delta y = -\frac{1}{2}gt^2$ | $\Delta y = (v_o \sin\theta)t - \frac{1}{2}gt^2$ |
| $v^2 = v_o{}^2 + 2a\Delta x$ | $\Delta t$ | $v_y{}^2 = -2g\Delta y$ | $v_y{}^2 = v_o{}^2 \sin^2\theta - 2g\Delta y$ |

- For an object in free fall, the object stops accelerating when the force of air resistance, $\vec{F}_{Air}$, equals the weight, $\vec{W}$. The object has reached its maximum velocity, the ***terminal velocity***.
- When a quarterback throws a football, the angle for a high, lob pass is related to the angle for a low, bullet pass. When both footballs are caught by a receiver standing in the same place, the sum of the launch angles is 90$^o$.
- In distance contests for projectiles launched by cannons, catapults, trebuchets, and similar devices, projectiles achieve the farthest distance when launched at a 45$^o$ angle.

- The ***range*** of a projectile launched at initial velocity $v_o$ and angle $\theta$ is $R = \left(\frac{v_o^2}{g}\right)\sin 2\theta$
- The ***maximum height*** of a projectile above its launch site is $y_{\max} = \frac{v_o^2 \sin^2\theta}{2g}$

## Chapter 5: Newton's Laws of Motion

**Table 5: Newton's Laws of Motion**

| | ***Modern Statement for Law*** | ***Translation*** |
|---|---|---|
| ***Newton's First Law:*** ***(Law of Inertia)*** Recall that mass is a measure of inertia. | If the net force on an object is zero, its velocity is constant. | An object at rest will remain at rest. An object in motion will remain in motion at constant velocity unless acted upon by an external force. |
| ***Newton's Second Law:*** | An object of mass $m$ has an acceleration $\vec{a}$ given by the net force $\sum \vec{F}$ divided by $m$. That is $\vec{a} = \frac{\sum \vec{F}}{m}$ | $F_{net} = ma$ |
| ***Newton's Third Law:*** Recall action-reaction pairs | For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction. | For every action, there is an equal but opposite reaction. |

### *A. Survey of Forces*

- A force is a push or a pull. The unit of force is the Newton (N); 1 N = 1 kg-m/s$^2$
- See Newton's laws of motion in Table 5. Common forces on a moving object include an applied force, a frictional force, a weight, and a normal force.
- ***Contact forces*** are action-reaction pairs of forces produced by physical contact of two objects. ***Review calculations regarding contact forces between two or more boxes.***
- ***Field forces*** like gravitational forces, electrostatic forces, and magnetic forces do not require direct contact. They are studied in later chapters.
- Forces on objects are represented in free-body diagrams. They are drawn with the tails of the vectors originating at an object's center of mass.
- ***Weight***, $\vec{W}$, is the gravitational force exerted by Earth on an object whereas mass, $m$, is a measure of the quantity of matter in an object ($W = mg$). Mass does not depend on gravity.
- ***Apparent weight***, $\vec{W}_a$, is the force felt from contact with the floor or a scale in an accelerating system. For example, the sensation of feeling heavier or lighter in an accelerating elevator.
- The ***normal force***, $\vec{N}$, is perpendicular to the contact surface along which an object moves or is capable of moving. Thus, for an object on a level surface, $\vec{N}$ and $\vec{W}$ are equal in size but opposite in direction. However, for an object on a ramp, this statement is not true because $\vec{N}$ is perpendicular to the surface of the ramp.
- ***Tension***, $\vec{T}$, is the force transmitted through a string. The tension is the same throughout the length of an ideal string.

- The ***force of an ideal spring*** stretched or compressed by an amount $x$ is given by ***Hooke's Law***, $\vec{F}_x = -kx$. Note that if we are only interested in magnitude, we use $F = kx$ where k is the spring or force constant. Hooke's Law is also used for rubber bands, bungee cords, etc.

## Chapter 6: Applications of Newton's Laws

### *A. Friction*

- Coefficient of ***static friction*** = $\mu_S = \frac{F_{S,\max}}{N}$ where $\vec{F}_{S,\max}$ is the max. force due to static friction.
- Coefficient of ***kinetic friction*** = $\mu_K = \frac{F_K}{N}$ where $\vec{F}_K$ is the force due to kinetic friction.
- A common lab experiment involves finding the angle at which an object just begins to slide down a ramp. In this case, a simple expression can be derived to determine the coefficient of static friction: $\mu_S = \tan\theta$. Note that this expression is independent of the mass of the object.

### *B. Newton's Second Law Problems (Includes Ramp Problems)*

1. Draw a free-body diagram to represent the problem.
2. If the problem involves a ramp, rotate the x- and y-axes so that the x-axis corresponds to the surface of the ramp.
3. Construct a vector table including all of the forces in the free-body diagram. For the vector table's column headings, use ***vector***, ***x-direction***, and ***y-direction***.
4. Determine the column total in each direction:
   a. If the object moves in that direction, the total is $ma$.
   b. If the object does not move in that direction, the total is zero.
   c. Since this is a Newton's Second Law problem, no other choices besides zero and $ma$ are possible.
5. Write the math equations for the sum of the forces in the x- and y-directions, and solve the problem. It is often helpful to begin with the y-direction since useful expressions are derived that are sometimes helpful later in the problem. Recall that the math equations regarding friction and weight are often substituted into the math equations to help solve the problem.

### *C. Equilibrium*

- An object is in translational equilibrium if the net force acting on it is zero, $\sum \vec{F} = 0$.
- Equivalently, an object is in equilibrium if it has zero acceleration.
- If a vector table is needed for an object in equilibrium, then $\sum \vec{F}_x = 0$ and $\sum \vec{F}_y = 0$.
- Typical problems involve force calculations for objects pressed against walls and tension calculations for pictures on walls, laundry on a clothesline, hanging baskets, pulley systems, traction systems, connected objects, etc.

### *D. Connected Objects*

- Connected objects are linked physically, and thus, they are also linked mathematically. For example, objects connected by strings have the same magnitude of acceleration.
- When a pulley is involved, the x-y coordinate axes are often rotated around the pulley so that the objects are connected along the x-axis.
- A classic example of a connected object is an Atwood's Machine, which consists of two masses connected by a string that passes over a single pulley. The acceleration for this system is given by $a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$.

## Chapter 7: Work and Kinetic Energy

### *A. Work*

- A force exerted through a distance performs mechanical work.
- When force and distance are parallel, $W = Fd$ with Joules (J) or Nm as the unit of work.
- When force and distance are at an angle, only the ***component*** of force in the direction of motion is used to compute the work: $W = (F\cos\theta)d = Fd\cos\theta$
- Work is negative if the force opposes the motion ($\theta$>90°). Also, 1 J = 1 Nm = 1 kg-m$^2$/s$^2$.
- If more than one force does the work, then $W_{Total} = \sum_{i=1}^{n} W_i$
- The work-kinetic energy theorem states that $W_{Total} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$
- See Table 6 for more information about kinetic energy.
- In thermodynamics, $W = Fd = Fd\left(\frac{A}{A}\right) = \left(\frac{F}{A}\right)\left(Ad\right) = P\Delta V$ for work done on or by a gas.

**Table 6: Kinetic Energy**

| *Kinetic Energy Type* | *Equation* | *Comments* |
|---|---|---|
| ***Kinetic Energy as a Function of Motion:*** | $K = \frac{1}{2}mv^2$ | Used to represent kinetic energy in most conservation of mechanical energy problems. |
| ***Kinetic Energy as a Function of Temperature:*** | $\left(\frac{1}{2}mv^2\right)_{ave} = K_{ave} = \frac{3}{2}kT$ | Kinetic theory relates the average kinetic energy of the molecules in a ***gas*** to the Kelvin temperature of the ***gas***. |

### *B. Determining Work from a Plot of Force Versus Position*

- In a plot of force versus position, work is equal to the area between the force curve and the displacement on the x-axis. For example, work can be easily computed using $W = Fd$ when rectangles are present in the diagram.
- For the case of a spring force, the work to stretch or compress a distance $x$ from equilibrium is $W = \frac{1}{2}kx^2$. On a plot of force versus position, work is the area of a triangle with base $x$ (displacement) and height $kx$ (magnitude of force using Hooke's Law, $F = kx$).

### *C. Determining Work in a Block and Tackle Lab*

- The experimental work done against gravity, $W_{Load}$, is the same as the theoretical work done by the spring scale, $W_{Scale}$.
- $W_{Output} = W_{Load} = Fd_{Load} = \vec{W}d_{Load} = mgd_{Load}$ where $d_{Load}$ = distance the load is raised.
- $W_{Input} = W_{Scale} = Fd_{Scale}$ where $F$ = force read from the spring scale and $d_{Scale}$ = distance the scaled moved from its original position.
- Note that the force read from the scale is ½ of the weight when two strings are used for the pulley system, and the force read is ¼ of the weight when four strings are used.

***D. Power***

- $P = \frac{W}{t}$ or $P = Fv$ with Watts (W) as the unit of Power.
- 1 W = 1 J/s and 746 W = 1 hp where hp is the abbreviation for horsepower.

## Chapter 8: Potential Energy and Conservation of Energy

### *A. Conservative Forces Versus Nonconservative Forces*

1. Conservative Forces
   - A conservative force does ***zero total work*** on any closed path. In addition, the work done by a conservative force in going from point A to point B is ***independent of the path*** from A to B. In other words, we can use the conservation of mechanical energy principle to solve complex problems because the problems only depend on the initial and final states of the system.
   - In a conservative system, the total mechanical energy remains constant: $E_i = E_f$. Since $E = U + K$, it follows that $U_i + K_i = U_f + K_f$. ***See Table 6 for kinetic energy, $K$, and Table 7 for potential energy, $U$, for additional information.***
   - For a ball thrown upwards, describe the shape of the kinetic energy, potential energy, and total energy curves on a plot of energy versus time.
   - Examples of conservative forces are gravity and springs.
2. Nonconservative Forces
   - The work done by a nonconservative force on a closed path is ***not*** zero. In addition, the work ***depends on the path*** going from point A to point B.
   - In a nonconservative system, the total mechanical energy is ***not*** constant. The work done by a nonconservative force is equal to the change in the mechanical energy of a system: $W_{Nonconservative} = W_{nc} = \Delta E = E_f - E_i$.
   - Examples of nonconservative forces include friction, air resistance, tension in ropes and cables, and forces exerted by muscles and motors.

**Table 7: Potential Energy**

| ***Potential Energy Type*** | ***Equation*** | ***Comments*** |
|---|---|---|
| ***Gravitational Potential Energy:*** | $U = mgh$ | Good approximation for an object near sea level on the Earth's surface. |
| ***Gravitational Potential Energy Between Two Point Masses:*** | $U = -G\frac{m_1 m_2}{r}$ where $G =$ 6.67 x $10^{-11}$ $Nm^2/kg^2$ = Universal Gravitation Constant | Works well at any altitude or distance between objects in the universe; recall that $r$ is the distance between the ***centers*** of the objects. |
| ***Elastic Potential Energy:*** | $U = \frac{1}{2}kx^2$ where $k$ is the force (spring) constant and $x$ is the distance the spring is stretched or compressed from equilibrium. | Useful for springs, rubber bands, bungee cords, and other stretchable materials. |

## Chapter 9: Linear Momentum and Collisions

### *A. Momentum*

- Linear momentum is given by $\vec{p} = m\vec{v}$ with kg-m/s as the unit of momentum.
- In a system having several objects, $\vec{p}_{Total} = \sum_{i=1}^{n} \vec{p}_i$.
- Newton's second law can be expressed in terms of momentum. The net force acting on an object is equal to the rate of change in its momentum: $\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t}$
- When mass is constant, $\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t} = \frac{m\Delta \vec{v}}{\Delta t} = m\vec{a}$.
- The impulse-momentum theorem states that $\vec{I} = \vec{F}_{ave}\Delta t = \Delta \vec{p}$ where the quantity $\vec{F}_{ave}\Delta t$ is called the impulse, $I$. For problem-solving purposes, a more useful form of the impulse-momentum theorem is $F\Delta t = m(v_f - v_i)$.
- A practical application of the impulse-momentum theorem, $\vec{F}_{ave}\Delta t = \Delta \vec{p}$, involves auto air bags. In an automobile collision, the change in momentum, $\Delta p$, remains constant. Thus, an increase in collision time, $\Delta t$, will result in a decreased force of impact, $\vec{F}$, reducing personal injury.

### *B. Collisions*

- For a system of objects, the conservation of linear momentum principle states that the net momentum is conserved if the net external force acting on the system is zero. In other words, $p_i = p_f$.
- For an elastic collision, $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ and for a perfectly inelastic collision, $m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$. See the collision types in Table 8.
- In an elastic collision in one dimension where mass $m_1$ is moving with an initial velocity $v_o$, and mass $m_2$ is initially at rest, the velocities of the masses after the collision are: $v_{1,f} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_o$ and $v_{2,f} = \left( \frac{2m_1}{m_1 + m_2} \right) v_o$.

**Table 8: Collision Types**

| *Collision Type and Example* | *Do the Objects Stick Together?* | *Is Momentum Conserved?* | *Is Kinetic Energy Conserved?* |
|---|---|---|---|
| ***Elastic:*** No permanent deformation occurs; billiard balls collide. | No | Yes | Yes |
| ***Inelastic:*** Permanent deformation occurs; Most automobile collisions. | No | Yes | No |
| ***Perfectly Inelastic:*** Permanent deformation occurs and objects lock together moving as a single unit; train cars collide and lock together. | Yes | Yes | No |

**Chapter 10: Rotational Kinematics and Energy**

***A. Rotational Motion***

- Angular position, $\Delta\theta$, in radians is given by $\Delta\theta = \frac{\Delta s}{r}$ where $\Delta s$ is arc length and $r$ is radius.
- Recall that $\theta(rad) = \left(\frac{2\pi\ radians}{360^o}\right)\theta(\deg)$.
- Counterclockwise (CCW) rotations are positive, and clockwise (CW) rotations are negative.
- In rotational motion, there are two types of speeds (angular speed and tangential speed) and three types of accelerations (angular acceleration, tangential acceleration, and centripetal acceleration). See Table 9 for a comparison.
- Since velocity is a vector, there are two ways that an acceleration can be produced: (1) changing the velocity's magnitude and (2) changing the velocity's direction. In centripetal acceleration, the velocity's ***direction*** changes.
- When a person drives a car in a circle at constant speed, the car has a centripetal acceleration due to its changing direction, but it has no tangential acceleration due to its constant speed.
- The total acceleration of a rotating object is the vector sum of its tangential and centripetal accelerations.

**Table 9: Comparing Angular and Tangential Speed and Angular, Tangential, and Centripetal Acceleration**

| ***Calculation*** | ***Equations*** | ***Units, Comments*** |
|---|---|---|
| ***Angular Speed:*** | $\omega_{ave} = \frac{\Delta\theta}{\Delta t}$ | • radians/s<br>• Same value for horses A and B, side-by-side on a merry-go-round. |
| ***Tangential Speed:*** | $v_t = r\omega$ | • m/s<br>• Different values for horses A and B, side-by-side on a merry-go-round. |
| ***Angular Acceleration:*** | $\alpha_{ave} = \frac{\Delta\omega}{\Delta t}$ | • radians/s$^2$<br>• Same value for horses A and B, side-by-side on a merry-go-round. |
| ***Tangential Acceleration:*** | $a_t = r\alpha$ | • m/s$^2$<br>• Different values for horses A and B, side-by-side on a merry-go-round. |
| ***Centripetal Acceleration:*** | $a_c = \frac{v_t^2}{r} = r\omega^2$ | • m/s$^2$<br>• $a_c$ is perpendicular to $a_t$ with $a_c$ directed toward the center of the circle and $a_t$ tangent to it. |

- Centripetal force, $F_C$, is a force that maintains circular motion: $F_C = ma_C = \frac{mv_t^2}{r} = mr\omega^2$
- The period, $T$, is the time required to complete one full rotation. If the angular speed is constant, then $T = \frac{2\pi}{\omega}$.
- The equations for rotational kinematics are the same as the equations for linear kinematics. See Table 10 for a comparison.

| **Table 10: Kinematic equations for Rotational Motion** | |
|---|---|
| ***Linear Equations*** | ***Angular Equations*** |
| $x = x_o + v_{ave}t$ | $\theta = \theta_o + \omega_{ave}t$ |
| $v = v_o + at$ | $\omega = \omega_o + \alpha t$ |
| $x = x_o + v_o t + \frac{1}{2}at^2$ | $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$ |
| $v^2 = v_o{}^2 + 2a\Delta x$ | $\omega^2 = \omega_o{}^2 + 2\alpha(\Delta\theta)$ |

- A comparison of linear and angular inertia, velocity, acceleration, Newton's second law, work, kinetic energy, and momentum are presented in Table 11.
- The moment of inertia, $I$, is the rotational analog to mass in linear systems. It depends on the shape or mass distribution of the object. In particular, an object with a large moment of inertia is difficult to start rotating and difficult to stop rotating. See Table 10-1 on p.298 for moments of inertia for uniform, rigid objects of various shapes and total mass.
- The greater the moment of inertia, the greater an object's rotational kinetic energy.
- An object of radius $r$, rolling without slipping, translates with linear speed $v$ and rotates with angular speed $\omega = \frac{v}{r}$.

| **Table 11: Comparing Equations for Linear Motion and Rotational Motion** | | |
|---|---|---|
| ***Measurement or Calculation*** | ***Linear Equations*** | ***Angular Equations*** |
| ***Inertia:*** | Mass, m | $I = kmr^2$ where $k =$ a constant |
| ***Average Velocity:*** | $v_{ave} = \dfrac{\Delta x}{\Delta t}$ | $\omega_{ave} = \dfrac{\Delta \theta}{\Delta t}$ |
| ***Average Acceleration:*** | $a_{ave} = \dfrac{\Delta v}{\Delta t}$ | $\alpha_{ave} = \dfrac{\Delta \omega}{\Delta t}$ |
| ***Newton's Second Law:*** | $F_{net} = ma$ | $\tau = I\alpha = F_{\perp} r$ |
| ***Work:*** | $W = Fd\cos\theta$ | $W = \tau\theta$ |
| ***Kinetic Energy:*** | $K = \dfrac{1}{2}mv^2$ | $K = \dfrac{1}{2}I\omega^2$ |
| ***Momentum:*** | $p = mv$ | $L = I\omega$ |

## Chapter 11: Rotational Dynamics and Static Equilibrium

### *A. Torque*

- A tangential force, $F$, applied at a distance, $r$, from the axis of rotation produces a torque $\tau = rF$ in Nm. (Since $F$ is perpendicular to $r$, this is sometimes written as $\tau = F_{\perp} r$.)
- A force applied at an angle to the radial direction produces the torque $\tau = rF\sin\theta$
- Counterclockwise torques are positive, and clockwise torques are negative.
- The rotational analog of force, $F = ma$, is torque, $\tau = I\alpha$, where $I =$ moment of inertia and $\alpha =$ angular acceleration.
- The conditions for an object to be in static equilibrium are that the total force and the total torque acting on the object must be zero: $\sum F_x = 0$, $\sum F_y = 0$, $\sum \tau = 0$. Related problems often involve bridges, scaffolds, signs, and rods held by wires.

### *B. Solving Static Equilibrium Problems*

1. Construct a diagram showing all of the forces.
2. Create an equation adding the forces together.
    - Remember to enter correct signs in your force equation. For example, upward forces are positive and downward forces are negative.
    - Since the object is not moving, set the force equation equal to zero. ($\sum F = 0$)

3. Create an equation adding the torques together.
   - If you are not sure what to do:
     - Start by writing the forces again leaving some space between them.
     - Multiply each force by an appropriate distance.
     - Each force and distance pair will have the same subscript.
   - Choose an axis of rotation in your diagram. It is helpful if your choice eliminates one of the two unknown forces. Then you can solve for the other force.
   - Remember to enter correct signs in your torque equation. Consider whether each force will create a counterclockwise or clockwise rotation resulting in a positive or negative torque, respectively.
   - Since the object is not moving, set the torque equation equal to zero. ($\sum \tau = 0$)
4. Substitute numbers in your torque equation and solve for the unknown force.
5. Substitute the force you determined in the last step into the force equation and solve for the other unknown force.

## ***C. Angular Momentum***

- The rotational analog of momentum, $p = mv$, is angular momentum, $L = I\omega$ in kg-m$^2$/s, where $I$ = moment of inertia and $\omega =$ angular velocity.
- If the net external torque acting on a system is zero, its angular momentum is conserved and $L_i = L_f$.

## ***D. Simple Machines***

- All machines are combinations or modifications of six fundamental types of machines called ***simple machines.***
- Simple machines include the lever, inclined plane, wheel and axle, wedge, pulley(s), and screw.
- Mechanical advantage, $MA$, is defined as $MA = \frac{F_{out}}{F_{in}} = \frac{d_{in}}{d_{out}}$. It is a number describing how much force or distance is multiplied by a machine.
- Efficiency is a measure of how well a machine works, and $\%\ Efficiency$ is calculated using $\%\ Efficiency = \left(\frac{W_{out}}{W_{in}}\right)(100)$ where $W_{out}$ is the work output and $W_{in}$ is the work input.

## **Chapter 12: Gravity**

### ***A. General Concepts About Gravity and Kepler's Laws***

- ***Newton's Law of Universal Gravitation*** shows that the force of gravity between two point masses, $m_1$ and $m_2$, separated by a distance $r$ is $F = G\frac{m_1 m_2}{r^2}$ where G is the universal gravitation constant, $G = 6.67\text{x}10^{-11}$ Nm$^2$/kg$^2$. ***Remember that $r$ is the distance between the centers of the point masses.***
- In Newton's Law of Universal Gravitation, notice that the force of gravity decreases with distance, $r$, as $\frac{1}{r^2}$. This is referred to as an ***inverse square dependence***.
- The ***superposition principle*** can be applied to gravitational force. If more than one mass exerts a gravitational force on a given object, the net force is simply the vector sum of each individual force. (The superposition principle is also used for electrostatic forces, electric fields, electric potentials, electric potential energies, and wave interference.)

- Replacing the Earth with a point mass at its center, the acceleration due to gravity at the surface of the Earth is given by $g = \frac{GM_{Earth}}{R_{Earth}^2}$. A similar equation is used to calculate the acceleration due to gravity at the surface of other planets and moons in the Solar System.
- At some altitude, $h$, above the Earth, $g_h = G\frac{M_{Earth}}{\left(R_{Earth} + h\right)^2}$ can be used to calculate the acceleration due to gravity.
- In 1798, Henry Cavendish first determined the value of G, which allowed him to calculate the mass of the Earth using $M_{Earth} = \frac{gR_{Earth}^2}{G}$.
- Using Tycho Brahe's observations concerning the planets, Johannes Kepler formulated three laws for orbital motion as shown in Table 12. Newton later showed that each of Kepler's laws follows as a direct consequence of the universal law of gravitation.
- As previously mentioned in the potential energy table, the gravitational potential energy, $U$, between two point masses $m_1$ and $m_2$ separated by a distance $r$ is $U = -G\frac{m_1 m_2}{r}$. This equation is used in mechanical energy conservation problems for astronomical situations.
- Energy conservation considerations allow the escape speed to be calculated for an object launched from the surface of the Earth: $v_e = \sqrt{\frac{2GM_{Earth}}{R_{Earth}}}$ = 11,200 m/s ≈ 25,000 mi/h.

**Table 12: Kepler's Laws of Orbital Motion**

| ***Law*** | ***Modern Statement for Law*** | ***Alternate Description*** |
|---|---|---|
| **1st Law:** | Planets follow elliptical orbits, with the Sun at one focus of the ellipse. | The paths of the planets are ellipses, with the center of the Sun at one focus. |
| **2nd Law:** | As a planet moves in its orbit, it sweeps out an equal amount of area in an equal amount of time. | An imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals. |
| **3rd Law:** | The period, $T$, of a planet increases as its mean distance from the Sun, $r$, raised to the 3/2 power. That is, $T = \left(\frac{2\pi}{\sqrt{GM_S}}\right)r^{3/2} = (\text{constant})\, r^{3/2}$ | The square of a planet's period, $T^2$, is proportional to the cube of its radius, $r^3$. That is, $T^2 = \frac{4\pi^2}{GM_S}r^3 = (\text{constant})\, r^3$ where $M_S$ = mass of the Sun. |